

Radiative corrections in nucleon time-like form factors measurements

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Abstract

The completely general radiative corrections to lowest order, including the final and initial state radiations, are studied in proton-antiproton annihilation to an electron-positron pair. Numerical estimates have been made in a realistic configuraton of the PANDA (FAIR) detector for the proton time-like form factors measurements.

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I. INTRODUCTION

Precise polarization measurements of the proton electromagnetic form factor [1] confirms the Q^2 dependence up to $Q^2 = 8.5 \text{ GeV}^2$, of the ratio $R = \mu G_E/G_M$ (where μ is the proton's magnetic moment and G_E and G_M are the electric and magnetic proton form factors) showing an approximately linear decrease of R with Q^2 . This fact is in disagreement with results obtained from a new Rosenbluth cross section measurement [2] and suggests that the source of the discrepancy is not simply experimental. Recently, there has been a revival of interest in this subject [3]. Some theoretical works [4-6] have investigated the two-photon exchange corrections to the lowest order QED. This effect has been shown to resolve partially the discrepancy [3,7]. It is well established that the Rosenbluth method is much more sensitive to the radiative corrections than the polarization method. Until now, intense theoretical activities to evaluate the radiative corrections to elastic electron-proton scattering, which include higher order radiative corrections [8,9] or model-dependent box diagram calculation [10], incorporating nucleon's substructure, are not able to make a definitive conclusion on this discrepancy. We should note that two-photon exchange corrections are small in general and at a 1% level for a large class of experiments [3]. With this renewal of interest, the importance of theoretical descriptions of nucleon form factors in the space-like and also in the time-like region is emphasized.

In principle, time-like form factors could be evaluated from the space-like equivalents by means of dispersion relations. The ratio between electric and magnetic proton form factors by using space-like and time-like data was recently analysed in the framework of dispersion relations [11,12]. However, all the published data in the time-like region [13-16] assumed $G_E = G_M$ to hold for all Q^2 and not only at threshold. As accurate data at high energy is lacking, and also to resolve the discrepancy between the LEAR [13] and BaBar [16] data observed close to threshold, the measurement of the proton form factors in the time-like region is planned at PANDA at FAIR in proton antiproton annihilation to an electron positron pair with unprecedented high accuracy [17].

As mentioned above, the radiative corrections could be correctly evaluated in the kinematical configuration of the scattering experiment to extract the physics observables of interest. This paper is devoted to a theoretical investigation of the $\bar{p}p \rightarrow e^+e^-$ process including radiative corrections to lowest order of perturbation theory. Among the recent papers devoted to this subject, one should mention [18] where the possibility to measure the charge asymmetry is presented. The charge-odd part presented in the differential cross section is the origin of this asymmetry. It is surprising that the authors [18] do not evaluate the radiative correction to the cross section with an energy cut on the (e^+e^-) invariant mass spectrum corresponding to the energy resolution of the PANDA detector. Recently, the authors of [19] reevaluate this correction in the laboratory frame, omitting the contribution of the hard photon as well as the contribution from the initial state radiation.

The first results of full simulations with PANDA detector [17] show the precisions of the cross section measurement of the order of 3%-5% can be obtained. In this context, we need to evaluate the radiative correction due to the final state radiation, namely the photon emitted from the electron or the positron, as well as the radiation in the initial state at the proton vertex, reduced due to the large mass of the proton (antiproton) compared to the electron one. In this paper, the complete general radiative corrections to lowest order, to the $\bar{p}p \rightarrow e^+e^-$ channel are investigated, in the kinematical configuration of the planned experiment at FAIR with the PANDA detector.

II. ELECTROMAGNETIC NUCLEON CURRENT OPERATOR AND BORN CROSS SECTION

Let us first introduce our notations and the definition of the electromagnetic nucleon current operator with the magnetic and electric form factors of the nucleon. For the annihilation process, in the one photon exchange approximation :

$$\bar{p}(p^-) + p(p^+) \rightarrow e^+(q^+) + e^-(q^-)$$

The corresponding Feynman diagram for this reaction is given in Fig.1.

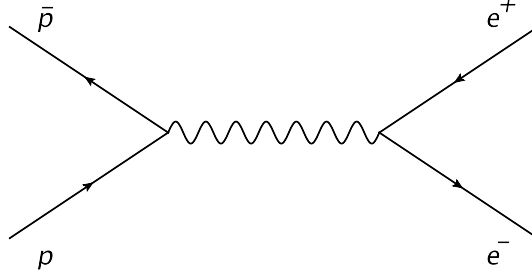


FIG. 1. One photon exchange diagram for the process $\bar{p}p \rightarrow e^+e^-$.

The Born amplitude has the form

$$M^B = \frac{4\pi\alpha}{s} \bar{v}(p^-) \Gamma_\mu u(p^+) \bar{u}(q^-) \gamma^\mu v(q^-) \quad (1)$$

with

$$\Gamma_\mu = F_1(s) \gamma_\mu + \frac{F_2(s)}{4M} [\gamma_\mu, \not{q}] \quad (2)$$

where the magnetic and electric form factors are related to the Dirac and Pauli form factors F_1 and F_2 :

$$\begin{aligned} G_E(s) &= F_1(s) + \tau F_2(s) \\ G_M(s) &= F_1(s) + F_2(s) \end{aligned} \quad (3)$$

$$s = q^2 = (p^- + p^+)^2, \quad \tau = s/(4M^2), \quad \beta_p^2 = 1 - 4M^2/s$$

M and m are respectively the proton and the electron masses.

The differential cross section in the Born approximation has the form :

$$\left[\frac{d\sigma}{d\Omega} \right]_B = \frac{\alpha^2}{4s\beta_p} \{ |G_M(s)|^2 (1 + \cos^2 \theta) + (1 - \beta_p^2) |G_E(s)|^2 \sin^2 \theta \} \quad (4)$$

Where θ is the scattering angle of the positron in the center of mass system (cms). This cross section was first derived by the authors of [20]. In the particular case, where the proton is considered as a pointlike particle, with $G_M(s) = G_E(s) = 1$, the formula (4) reduces to

$$\left[\frac{d\sigma}{d\Omega} \right]_B^0 = \frac{\alpha^2}{4s\beta_p} \{ (2 - \beta_p^2) \sin^2 \theta \} \quad (5)$$

III. QED RADIATIVE CORRECTIONS TO FIRST ORDER

The proton electromagnetic form factors can be extracted from the angular distribution of the final lepton in the Born cross section. However, this distribution is altered from its zeroth-order shape by radiative corrections. In practice, the distorted distribution by radiative effects can be written as :

$$\left[\frac{d\sigma}{d\Omega}\right]_R = \left[\frac{d\sigma}{d\Omega}\right]_B (1 + \delta) \quad (6)$$

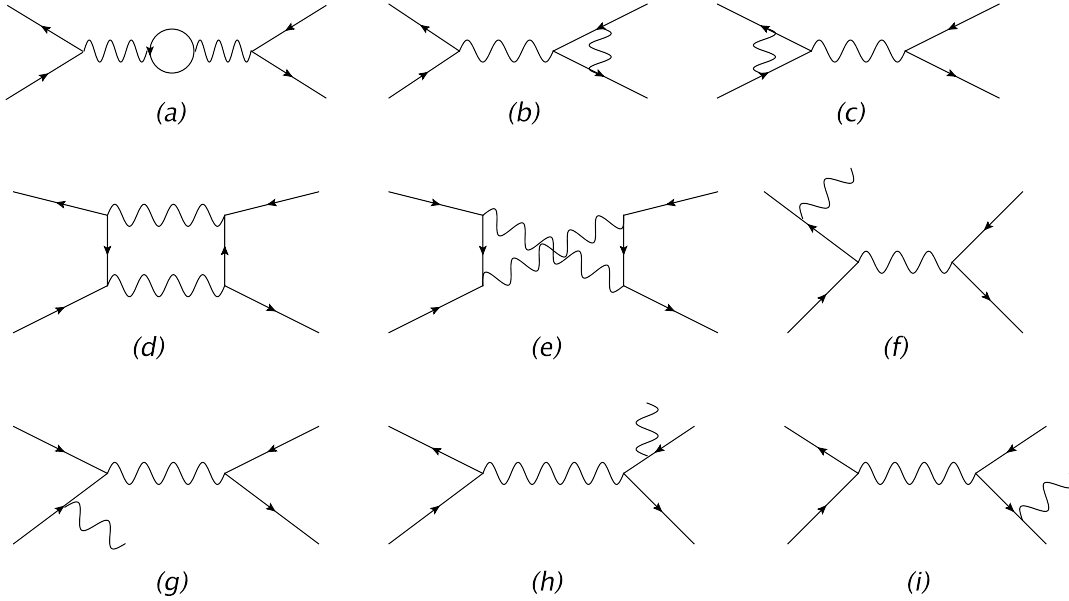


FIG. 2. Feynman diagrams for the first-order radiative correction in $\bar{p}p \rightarrow e^+e^-$. Note that the vacuum polarization loop (a) includes all fermions

The set of diagrams contributing to the first order corrections are shown in Fig. 2. The virtual correction comes from the interference between the born diagram (Fig. 1) and the diagrams (a)-(e) of Fig. 2. The bremsstrahlung from the initial state (diagrams (f) and (g)) alters the effective center of mass energy and significantly changes the kinematics of the final lepton pair. And finally, the photon emission from the final state is represented by the diagrams (h) and (i). Only the bremsstrahlung corrections lead to infrared singularities.

These singularities are cancelled order by order by virtual corrections. We adopt the standard treatment of the bremsstrahlung, separating the soft photon contribution with the emitted photon energy up to an infrared cut-off parameter ω where the soft photon approximation holds, and the hard photon contribution from ω up to an experimental cut depending on the energy resolution of the detector. This separation is somewhat arbitrary, so we have checked that the total radiative correction (Virtual+soft+hard) does not depend on this infrared cut-off.

The virtual and real photon corrections are achieved by the factorization of the cross section in Eq. (6) with

$$\delta = \delta_{SV} + \delta_H \quad (7)$$

We write down the soft and virtual correction together to remove the infrared singularities. The remaining term δ_{SV} (soft+virtual) is now finite. The hard photon contribution δ_H depends, of course, on the energy resolution of the detector. The full simulations in a realistic configuration of the detector will allow the determination of the experimental cut on the maximum energy of the real photon emitted or preferably, on the invariant mass spectrum of the final lepton pair.

Below we present the details of our investigation of the different contributions to the radiative corrections in $\bar{p}p \rightarrow e^+e^-$.

A. Virtual and soft photon contributions

In this section, following the previous work [18], we will suppose the proton to be the pointlike particle. In the range of energy and momentum transfers considered here ($4M^2 \leq s \leq 30 \text{ GeV}^2$), the correction due to the finite size of the nucleon is found in general to be much smaller than the other contributions [8]. The radiative factor δ is a ratio between the corrected cross section and the Born cross section where the form factors are both present. This could be one of the reasons, why the virtual radiative correction is practically independant of the model assumption on the form factors.

1. *Virtual correction :*

The Born cross section with one-loop corrections can be written in the form

$$\left[\frac{d\sigma}{d\Omega}\right]_{BV} = \left[\frac{d\sigma}{d\Omega}\right]_B^0 \left\{ \left| \frac{1}{1-\Pi} \right|^2 \left[1 + \frac{2\alpha}{\pi} (F_e^{(2)} + F_1^{(2)}) + \frac{4\alpha}{\pi} \frac{F_2^{(2)}}{2 - \beta_p^2 \sin^2 \theta} \right] + \frac{2\alpha}{\pi} \frac{I(t, u, s)}{s(2 - \beta_p^2 \sin^2 \theta)} \right\} \quad (8)$$

where s , t and u are the Mandelstam variables of the process, namely

$$s = (p^- + p^+)^2, \quad t = (p^- - q^+)^2, \quad u = (p^- - q^-)^2$$

and Ω is defined as the solid angle Ω_{e^+} of the positron in the center of mass system.

At lowest order in α , the vacuum correction can be written as

$$\left| \frac{1}{1-\Pi} \right|^2 \equiv 1 + \delta_{vac} \equiv 1 + \frac{\alpha}{\pi} F_{vac} \quad (9)$$

and keeping in the cross section, the correction in the same order in α , one gets

$$\left[\frac{d\sigma}{d\Omega}\right]_{BV} = \left[\frac{d\sigma}{d\Omega}\right]_B^0 \left\{ 1 + \frac{\alpha}{\pi} \left[F_{vac} + 2F_e^{(2)} + 2F_1^{(2)} + \frac{4F_2^{(2)}}{2 - \beta_p^2 \sin^2 \theta} + \frac{2I(t, u, s)}{s(2 - \beta_p^2 \sin^2 \theta)} \right] \right\} \quad (10)$$

$$F_{vertex}^e \equiv 2F_e^{(2)} \quad (11)$$

$$F_{vertex}^p \equiv 2F_1^{(2)} + \frac{4F_2^{(2)}}{2 - \beta_p^2 \sin^2 \theta} \quad (12)$$

$$F_{box} \equiv \frac{2I(t, u, s)}{s(2 - \beta_p^2 \sin^2 \theta)} \quad (13)$$

$$\delta_{vertex}^e = \frac{\alpha}{\pi} F_{vertex}^e \quad \delta_{vertex}^p = \frac{\alpha}{\pi} F_{vertex}^p \quad \delta_{box} = \frac{\alpha}{\pi} F_{box} \quad (14)$$

$$\left[\frac{d\sigma}{d\Omega}\right]_{BV} = \left[\frac{d\sigma}{d\Omega}\right]_B^0 \left\{ 1 + \delta_{vac} + \delta_{vertex}^e + \delta_{vertex}^p + \delta_{box} \right\} \quad (15)$$

The real parts of different contributions through the interference with the Born amplitude which is real itself, are given below.

For the vacuum polarisation contribution, the loop in diagram (a) of Fig.2 includes all

fermions, including the muon-loop and hadronic loop. The charged pion pair as hadron state is taken into account in the vacuum polarisation contribution [18].

$$\delta_{vac} = \frac{\alpha}{\pi} \left[2 \left(\Pi_{e^+e^-} + \Pi_{\mu^+\mu^-} + \Pi_{\pi^+\pi^-} \right) \right] \quad (16)$$

$$\Pi_{e^+e^-} = \frac{1}{3} \left(L_e - \frac{5}{3} \right) ; \quad L_e = \ln \frac{s}{m^2} \quad (17)$$

$$\Pi_{\mu^+\mu^-} = -\frac{8}{9} + \frac{\beta_\mu^2}{3} + \beta_\mu \left(\frac{1}{2} - \frac{1}{6} \beta_\mu^2 \right) L_\mu \quad (18)$$

$$\beta_\mu = \sqrt{1 - \frac{4 M_\mu^2}{s}} \quad L_\mu = \ln \frac{1 + \beta_\mu}{1 - \beta_\mu} \quad (19)$$

$$\Pi_{\pi^+\pi^-} = 2 \left[\frac{1}{12} L_\pi - \frac{2}{3} - 2 \beta_\pi^2 \right] \quad (20)$$

$$\beta_\pi = \sqrt{1 - \frac{4 M_\pi^2}{s}} \quad L_\pi = \ln \frac{1 + \beta_\pi}{1 - \beta_\pi} \quad (21)$$

At the lepton vertex, the expression of δ_{vertex}^e was derived long time ago by the authors of [21]. To deal with the infrared divergent term, we consider the extra virtual photon of the process in Fig. 2 with a mass λ .

$$F_{vertex}^e = 2 \left[\left(\ln \frac{m}{\lambda} - 1 \right) (1 - L_e) - \frac{1}{4} L_e - \frac{1}{4} L_e^2 + \frac{\pi^2}{3} \right] \quad (22)$$

Isolating explicitly the divergent term from eq. (22), one obtains :

$$F_{vertex}^e = 2 (L_e - 1) - \frac{1}{2} L_e - \frac{1}{2} L_e^2 + \frac{2 \pi^2}{3} + 2 (1 - L_e) \ln \frac{m}{\lambda} \quad (23)$$

$$\delta_{vertex}^e = \frac{\alpha}{\pi} \left[2 (L_e - 1) - \frac{1}{2} L_e - \frac{1}{2} L_e^2 + \frac{2 \pi^2}{3} + 2 (1 - L_e) \ln \frac{m}{\lambda} \right] \quad (24)$$

At the hadron vertex, we assume the proton to be a pointlike particle and we get :

$$F_{vertex}^p = 2 F_1^{(2)} + \frac{4 F_2^{(2)}}{2 - \beta_p^2 \sin^2 \theta} \quad (25)$$

$$2 F_1^{(2)} = 2 \left\{ -1 - \frac{1}{4\beta_p} L_{\beta_p} + \frac{1+\beta_p^2}{2\beta_p} \left[\frac{\pi^2}{3} + L_{\beta_p} + Sp\left(\frac{1-\beta_p}{1+\beta_p}\right) - \frac{1}{4} L_{\beta_p}^2 - L_{\beta_p} \ln \frac{2\beta_p}{1+\beta_p} \right] + \left(1 - \frac{1+\beta_p^2}{2\beta_p} L_{\beta_p}\right) \ln \frac{M}{\lambda} \right\} \quad (26)$$

$$\frac{4 F_2^{(2)}}{2 - \beta_p^2 \sin^2 \theta} = -\frac{1 - \beta_p^2}{\beta_p} \frac{L_{\beta_p}}{2 - \beta_p^2 \sin^2 \theta} \quad (27)$$

$$L_{\beta_p} \equiv \ln \frac{1 + \beta_p}{1 - \beta_p} \quad (28)$$

The expression for δ_{box} due to the interference between the Born amplitude and the two-photon exchange diagrams (d) and (e) in Fig. 2 can be found in [18] and is given in Appendix A.

2. Soft photon contribution

The soft photon contribution comes from the reaction $\bar{p}p \rightarrow \gamma e^+ e^-$, where the energy k_0 of the emitted photon goes to zero. The soft photon cross section is related to the Born cross section using

$$d\sigma_{\text{soft}} = -\frac{\alpha}{2\pi^2} d\sigma_{\text{B}} I_{\text{soft}} \quad (29)$$

The soft photon contribution to the cross section reads

$$\delta_R = -\frac{\alpha}{2\pi^2} I_{\text{soft}} \quad (30)$$

with

$$I_{\text{soft}} = \int_0^{\omega'} \left(\frac{p^+}{k \cdot p^+} - \frac{p^-}{k \cdot p^-} + \frac{q^-}{k \cdot q^-} - \frac{q^+}{k \cdot q^+} \right)^2 \frac{d^3 \mathbf{k}}{2k_0} \quad (31)$$

$\omega' = \sqrt{\omega^2 - \lambda^2}$. λ is a virtual mass of the photon ($\lambda \rightarrow 0$)

$$I_{\text{soft}} = I_{\text{soft}}^{\text{e}} + I_{\text{soft}}^{\text{p}} + I_{\text{soft}}^{\text{e-p}} \quad (32)$$

$$I_{\text{soft}}^{\text{e}} = \int_0^{\omega'} \left(\frac{q^-}{k \cdot q^-} - \frac{q^+}{k \cdot q^+} \right)^2 \frac{d^3 \mathbf{k}}{2k_0} \quad (33)$$

$$I_{\text{soft}}^{\text{p}} = \int_0^{\omega'} \left(\frac{p^+}{k \cdot p^+} - \frac{p^-}{k \cdot p^-} \right)^2 \frac{d^3 \mathbf{k}}{2k_0} \quad (34)$$

$$I_{\text{soft}}^{\text{e-p}} = 2 \int_0^{\omega'} \left(\frac{p^+}{k \cdot p^+} - \frac{p^-}{k \cdot p^-} \right) \left(\frac{q^-}{k \cdot q^-} - \frac{q^+}{k \cdot q^+} \right) \frac{d^3 \mathbf{k}}{2k_0} \quad (35)$$

We adopt the 't Hooft and Veltman method [22] to evaluate these integrals, rewritten their final result, using our metric à la Bjorken and Drell. We give more details of this transposition in appendix B.

The electron term in the contribution of the soft photon emitted at the lepton vertex can be written as

$$I_{\text{soft}}^{\text{e}} = m^2 \mathcal{I}_{q^- q^-} + m^2 \mathcal{I}_{q^+ q^+} - 2 q^- \cdot q^+ \frac{\mathcal{L}_{q^- q^+}}{2} \quad (36)$$

with

$$\beta_e^2 = 1 - \frac{4m^2}{s} \quad 2 q^- \cdot q^+ = s - 2m^2 = \frac{s(1 + \beta_e^2)}{2} \quad (37)$$

The evaluation of the two first diagonal terms is straightforward and one gets :

$$m^2 \mathcal{I}_{q^- q^-} = m^2 \mathcal{I}_{q^+ q^+} = \pi \left[2 \ln \frac{2\omega}{\lambda} - \ln \frac{s}{m^2} \right] \quad (38)$$

The non-diagonal term is derived from the 't Hooft and Veltman method. We need to separate the finite term (finite) from the infrared singularity term (div) depending on λ .

$$\frac{\mathcal{L}_{q^- q^+}(\text{div})}{2} = \frac{2\pi}{s\beta_e} \ln \frac{(1 + \beta_e)^2}{(1 - \beta_e)^2} \ln \frac{2\omega}{\lambda} \quad (39)$$

$$\frac{\mathcal{L}_{q^- q^+}(\text{finite})}{2} = \frac{2\pi}{s\beta_e} \left[Sp \left(-\frac{2\beta_e}{1 - \beta_e} \right) - Sp \left(\frac{2\beta_e}{1 + \beta_e} \right) \right] \quad (40)$$

$$I_{\text{soft}}^{\text{e}} = -2\pi \left\{ \left[\frac{1 + \beta_e^2}{2\beta_e} \ln \frac{(1 + \beta_e)^2}{(1 - \beta_e)^2} - 2 \right] \ln \frac{2\omega}{\lambda} + \ln \frac{s}{m^2} + \frac{(1 + \beta_e^2)}{2\beta_e} \left[Sp \left(-\frac{2\beta_e}{1 - \beta_e} \right) - Sp \left(\frac{2\beta_e}{1 + \beta_e} \right) \right] \right\} \quad (41)$$

Now for the soft photon emitted at the hadron vertex, assuming the proton as a pointlike particle, $I_{\text{soft}}^{\text{p}}$ is derived from $I_{\text{soft}}^{\text{e}}$, replacing β_e by β_p :

$$I_{\text{soft}}^{\text{p}} = -2\pi \left\{ \left[\frac{1 + \beta_p^2}{2\beta_p} \ln \frac{(1 + \beta_p)^2}{(1 - \beta_p)^2} - 2 \right] \ln \frac{2\omega}{\lambda} + \ln \frac{s}{M^2} + \frac{(1 + \beta_p^2)}{2\beta_p} \left[Sp\left(-\frac{2\beta_p}{1 - \beta_p}\right) - Sp\left(\frac{2\beta_p}{1 + \beta_p}\right) \right] \right\} \quad (42)$$

Here again, we separate the interference term between the soft photon contribution from the lepton and hadron vertex into an infrared divergent term depending on the mass λ and a finite term :

$$I_{\text{soft}}^{\text{e-p}}(\text{div}) = 4\pi \ln \frac{M^2 - t}{M^2 - u} \ln \frac{M^2}{\lambda^2} \quad (43)$$

$$I_{\text{soft}}^{\text{e-p}}(\text{finite}) = 4\pi \left\{ 2 \ln \frac{M^2 - t}{M^2 - u} \ln \frac{2\omega}{M} + Sp\left(1 + \frac{(1 + \beta_p) s t}{2 M^4}\right) - Sp\left(1 + \frac{(1 + \beta_p) s u}{2 M^4}\right) + Sp\left(1 + \frac{s t}{(M^2 - t)^2}\right) - Sp\left(1 + \frac{s u}{(M^2 - u)^2}\right) + Sp\left(1 + \frac{(1 - \beta_p) s t}{2 M^4}\right) - Sp\left(1 + \frac{(1 - \beta_p) s u}{2 M^4}\right) \right\} \quad (44)$$

$Sp(x)$ is the dilogarithm or Spence's function defined as :

$$Sp(x) = - \int_0^x \frac{\ln(1 - t)}{t} dt$$

One can check that the infrared terms depending on λ disappear when we sum up the contributions from the virtual and soft photon corrections.

We show in table I and II, the soft and virtual corrections with final and initial state radiations. The superscript “e” or “e - p” indicate the corrections from the lepton final state

radiation or the total correction with final and initial state radiations. ω is the infrared cut-off on photon energy, separating the soft and hard photon contributions.

TABLE I. Soft and virtual corrections : $s = 5.4 \text{ GeV}^2$, $\omega = 12 \text{ MeV}$, $\omega/E_{e^+} \approx 1\%$

$\theta_{e^+} \setminus \delta$	δ_{vac}	δ_{vertex}^e	δ_R^e	δ_{vertex}^p	δ_R^p	δ_{box}	δ_R^{e-p}	δ_{SV}^e	δ_{SV}^{e-p}
30.	0.0103	-0.2602	-0.0147	0.0160	-0.0127	-0.0033	-0.0697	-0.2646	-0.3309
60.	0.0103	-0.2602	-0.0147	0.0147	-0.0127	-0.0020	-0.0365	-0.2646	-0.2991
90.	0.0103	-0.2602	-0.0147	0.0132	-0.0127	0.0000	0.0000	-0.2646	-0.2641
120.	0.0103	-0.2602	-0.0147	0.0147	-0.0127	0.0020	0.0365	-0.2646	-0.2261
150.	0.0103	-0.2602	-0.0147	0.0160	-0.0127	0.0033	0.0697	-0.2646	-0.1915

TABLE II. Soft and virtual corrections : $s = 12.9 \text{ GeV}^2$, $\omega = 18 \text{ MeV}$, $\omega/E_{e^+} \approx 1\%$

$\theta_{e^+} \setminus \delta$	δ_{vac}	δ_{vertex}^e	δ_R^e	δ_{vertex}^p	δ_R^p	δ_{box}	δ_R^{e-p}	δ_{SV}^e	δ_{SV}^{e-p}
30.	0.0135	-0.2991	-0.0006	0.0113	-0.0314	-0.0112	-0.1169	-0.2792	-0.4162
60.	0.0135	-0.2991	-0.0006	0.0095	-0.0314	-0.0059	-0.0558	-0.2792	-0.3668
90.	0.0135	-0.2991	-0.0006	0.0062	-0.0314	0.0000	0.0000	-0.2792	-0.3043
120.	0.0135	-0.2991	-0.0006	0.0095	-0.0314	0.0059	0.0558	-0.2792	-0.2452
150.	0.0135	-0.2991	-0.0006	0.0113	-0.0314	0.0112	0.1169	-0.2792	-0.1824

The radiative corrections given in the different columns are explained in the text. Let us make some comments on the radiative correction (soft+virtual) factor values in the two last columns.

- If only the final state radiation is taken into account, it is independent of the lepton scattering angle in the center of mass system. This is not the case for the initial state radiation, due to the interference between the initial and final state radiations given by δ_R^{e-p} .

- A remarkable feature is the asymmetry observed in the lepton angular distribution due to the charge-odd term. The angular distributions of the e^+ and e^- are different.
- The contribution of the box diagrams (two-photon exchange) is negligible, less than 1 %, in agreement with the hard rescattering mechanism calculation [23].

In principle, the hard photon contributions in the next section do not alter these features.

B. Hard photon emission

We consider next the contribution from hard photon emission :

$$\bar{p}(p^-) + p(p^+) \rightarrow e^+(q^+) + e^-(q^-) + \gamma(k)$$

The invariant mass W of the (e^+e^-) system is defined as :

$$W^2 = (q^+ + q^-)^2 = (p^- + p^+ - k)^2$$

In contrast with the previous subsection, the explicit proton form factors [24] are introduced in our estimation of the hard photon contribution. The correction factor δ_H is a ratio between the cross section with an extra real photon and the Born cross section and is not expected to be very sensitive to the explicit shape of the form factors.

We have performed the exact calculation of δ_H and the numerical results are displayed in Table III in terms of the total correction.

The amplitude is written as a sum of four amplitudes :

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4 \quad (45)$$

$\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4$ are respectively the amplitudes of the diagrams (h), (i), (f), (g) in Fig. 2. The expressions of different amplitudes are displayed in Appendix C.

$$\mathcal{M}(\lambda, m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) = \sum_i \frac{A_i^\sigma(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p)}{D_i} \varepsilon_\sigma^*(k, \lambda) \quad (46)$$

$$\mathcal{M}^*(\lambda, m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) = \sum_j \frac{A_j^{\sigma'^*}(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p)}{D_j} \varepsilon_{\sigma'}(k, \lambda) \quad (47)$$

$$\begin{aligned} & \sum_{\lambda} |\mathcal{M}(\lambda, m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p)|^2 \\ &= \sum_{ij} \frac{A_i^{\sigma}(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) A_j^{\sigma'^*}(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p)}{D_i D_j} \sum_{\lambda} \varepsilon_{\sigma}^*(k, \lambda) \varepsilon_{\sigma'}(k, \lambda) \\ &= - \sum_{ij} \frac{A_i^{\sigma}(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) A_{\sigma j}^*(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p)}{D_i D_j} \end{aligned} \quad (48)$$

$$X_{ij} \equiv - \sum_{m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p} A_i^{\sigma}(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) A_{\sigma j}^*(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) \quad (49)$$

$$\sum_{\lambda, m_{e^+}, \dots} |\mathcal{M}(\lambda, m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p)|^2 = \sum_{ij} \frac{X_{ij}}{D_i D_j} \quad (50)$$

$$\begin{aligned} & \sum_{\lambda, m_{e^+}, \dots} |\mathcal{M}(\lambda, m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p)|^2 \\ &= \frac{X_{11}}{D_1^2} + \frac{X_{22}}{D_2^2} + \frac{X_{33}}{D_3^2} + \frac{X_{44}}{D_4^2} + \frac{X_{12} + X_{21}}{D_1 D_2} + \frac{X_{13} + X_{31}}{D_1 D_3} + \frac{X_{14} + X_{41}}{D_1 D_4} \\ &+ \frac{X_{23} + X_{32}}{D_2 D_3} + \frac{X_{24} + X_{42}}{D_2 D_4} + \frac{X_{34} + X_{43}}{D_3 D_4} \end{aligned} \quad (51)$$

$$\frac{d^5\sigma}{dE_{\gamma} d\Omega_{\gamma} d\Omega_{e^+}} = \sum_{1 \leq i, j \leq 4} \left[\frac{d^5\sigma}{dE_{\gamma} d\Omega_{\gamma} d\Omega_{e^+}} \right]_{ij} \quad (52)$$

$$\left[\frac{d^2\sigma}{d\Omega_{e^+}} \right]_R(E_{\gamma}^{max}) = \left[\frac{d^2\sigma}{d\Omega_{e^+}} \right]_B(1 + \delta_{SV}(\omega)) + \int_{\omega}^{E_{\gamma}^{max}} \frac{d^5\sigma}{dE_{\gamma} d\Omega_{\gamma} d\Omega_{e^+}} dE_{\gamma} d\Omega_{\gamma} \quad (53)$$

$$\delta^e = \delta_{SV}^e(\omega) + \sum_{1 \leq i, j \leq 2} \int_{\omega}^{E_{\gamma}^{max}} \left[\frac{d^5\sigma}{dE_{\gamma} d\Omega_{\gamma} d\Omega_{e^+}} \right]_{ij} dE_{\gamma} d\Omega_{\gamma} \Big/ \left[\frac{d^2\sigma}{d\Omega_{e^+}} \right]_B \quad (54)$$

$$\delta^{e-p} = \delta_{SV}^{e-p}(\omega) + \sum_{1 \leq i, j \leq 4} \int_{\omega}^{E_{\gamma}^{max}} \left[\frac{d^5 \sigma}{dE_{\gamma} d\Omega_{\gamma} d\Omega_{e^+}} \right]_{ij} dE_{\gamma} d\Omega_{\gamma} \Big/ \left[\frac{d^2 \sigma}{d\Omega_{e^+}} \right]_B \quad (55)$$

The terms with the sum on i, j in formulas (54) and (55) are respectively the hard photon contributions δ_H^e and δ_H^{e-p} .

TABLE III. Total radiative corrections for $s = 5.4 \text{ GeV}^2$ (two left columns) and for $s = 12.9 \text{ GeV}^2$ (two right columns), assuming the energy of the hard photon emission up to 100 MeV

$\theta_{e^+} \setminus \delta$	δ^e δ^{e-p}		δ^e δ^{e-p}	
30.	-0.1154	-0.1502	-0.1517	-0.2388
60.	-0.1154	-0.1298	-0.1517	-0.1980
90.	-0.1154	-0.1080	-0.1517	-0.1616
120.	-0.1154	-0.0831	-0.1517	-0.1183
150.	-0.1154	-0.0598	-0.1517	-0.0725

The total radiative correction depends, of course, on the energy cut of the emitted photon. The full simulation with a realistic configuration of the PANDA detector could determine the precise value of this experimental cut.

IV. NUMERICAL RESULTS

The numerical results strongly depend on the experimental energy cut of the emitted photon. This dependence is displayed in Figs. 3-4 for two values of the antiproton energy. The distribution in Fig. 5 allows one to construct a Monte Carlo event generator for $\bar{p}p \rightarrow \gamma e^+ e^-$. The full simulation with a realistic environment of the PANDA detector is needed to determine the experimental cut on the invariant mass of the $(e^+ e^-)$ system.

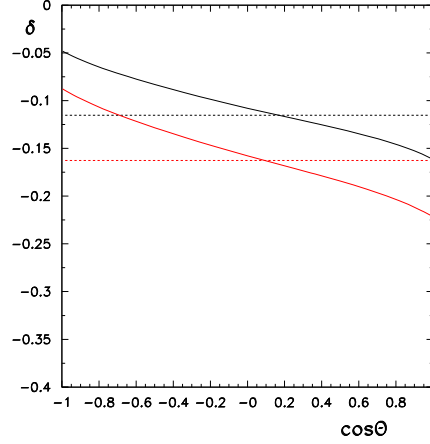


FIG. 3. (Color online) Total radiative corrections factor δ as a function of $\cos(\theta)$ in the CM frame, for $s = 5.4 \text{ GeV}^2$: for $E_\gamma^{max} = 100 \text{ MeV}$ (black line) and 50 MeV (red line). The corresponding dashed lines are only the final state radiation contribution

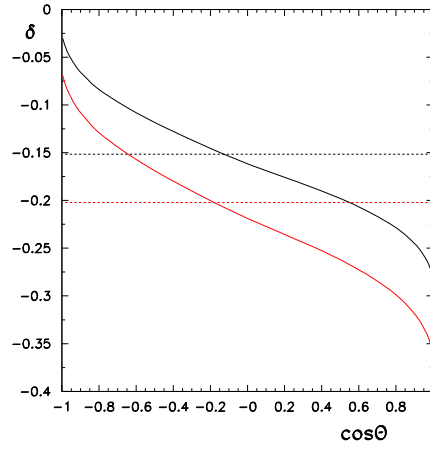


FIG. 4. (Color online) Total radiative corrections factor δ as a function of $\cos(\theta)$ in the CM frame, for $s = 12.9 \text{ GeV}^2$: for $E_\gamma^{max} = 100 \text{ MeV}$ (black line) and 50 MeV (red line). The corresponding dashed lines are only the final state radiation contribution

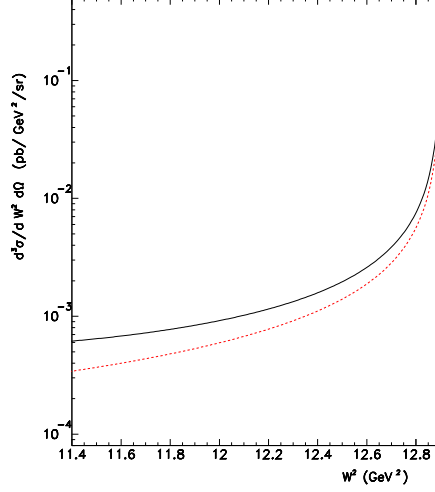


FIG. 5. (Color online) Differential cross section $d^2\sigma/dW^2 d\Omega_{e^+}$ as a function of W^2 (W is the invariant mass of the (e^+e^-) system), for $s = 12.9 \text{ GeV}^2$, $E_\gamma^{max} = 100 \text{ MeV}$ and $\theta = 30^\circ$ (black line). The corresponding red dashed line is only the final state radiation contribution

The corrected cross section is displayed for comparison with the Born cross section in Fig. 6. The different curves are obtained with one model [24] for the form factors. Of course the normalisation of these cross sections depend on the model assumption. One can also remark the asymmetry of the black line due to charge-odd term when the the initial state radiation at the hadron vertex is included. The measurement of this asymmetry term included in the angular distribution seems to be a difficult task. Fig. 7 shows the ratio of the correction factors, reflecting the importance of the initial state radiation versus final state radiation.

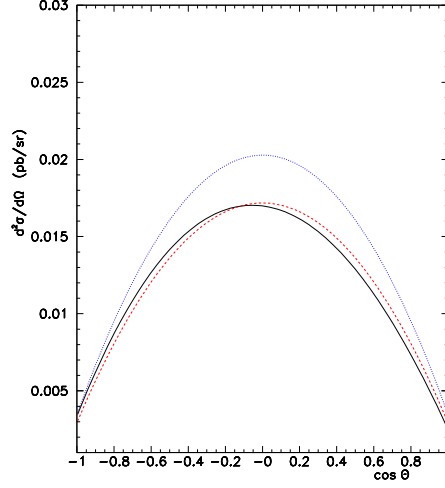


FIG. 6. (Color online) Corrected differential cross section $d^2\sigma/d\Omega_{e^+}$ as a function of $\cos(\theta)$ in the CM frame, for $s = 12.9 \text{ GeV}^2$, $E_\gamma^{max} = 100 \text{ MeV}$ (black line). The corresponding red dashed line is only the final state radiation contribution. The blue dotted line is the Born cross section with a model assumption of the form factors [24].

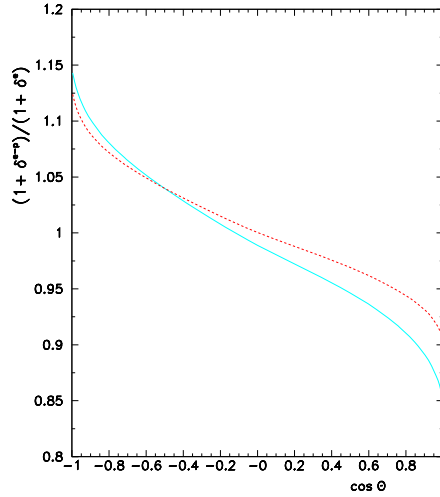


FIG. 7. (Color online) Ratio between the $1 + \delta^{e-p}$ and $1 + \delta^e$ as a function of $\cos(\theta)$ in the CM frame, for $s = 12.9 \text{ GeV}^2$: for $E_\gamma^{max} = 100 \text{ MeV}$ (cyan line) and 200 MeV (red dashed line)

The total radiative correction have an important symmetry that is worth to mention.

The values of δ are the corrections to the angular distribution of the positron (e^+). The corresponding distribution of the electron (e^-) is obtained by replacing (θ) by $(\pi - \theta)$ in order to respect the C-charge symmetry. We have checked numerically that the total radiative correction reads :

$$\delta^{(e^+)}(\theta) = \delta^{(e^-)}(\pi - \theta)$$

This observation led us to define two interesting observables namely :

$$\mathcal{S} = \frac{1}{2} \left(\frac{d\sigma}{d\Omega_{e^+}} + \frac{d\sigma}{d\Omega_{e^-}} \right)$$

$$\mathcal{A} = \left(\frac{d\sigma}{d\Omega_{e^+}} - \frac{d\sigma}{d\Omega_{e^-}} \right) \Bigg/ \left(\frac{d\sigma}{d\Omega_{e^+}} + \frac{d\sigma}{d\Omega_{e^-}} \right)$$

The first one contains the charge-even terms and is the corrected Born cross section for form factors extraction. The second one is the charge asymmetry observable due to the odd part. The value of this charge symmetry term \mathcal{A} is rather large ($\sim 5\%$) and can be measured with the PANDA detector. In contrast to the observable \mathcal{S} , the charge symmetry \mathcal{A} is more sensitive to the model assumption on the form factors due to the interference terms between the initial and final state radiations.

V. CONCLUSIONS

We conclude that the initial state radiation (at the hadron vertex) is not negligible and can be calculated and incorporated into the total radiative correction. If the precision of the cross section measurement is of the order of 3-5%, the total correction obtained in this paper can be used to correct the Born cross section before the comparison with the experimental data. We have shown that the interference between the initial and final state radiations is rather large. The evaluation of this correction in terms of structure functions and evolution equations [25] must be performed with caution. A study of the angular distribution of the photon shows a non negligible part of the photons emitted outside the cone of angle $\theta_\gamma \leq m/E$.

In contrast to space-like elastic electron-proton scattering, in the time-like region we can consider the shape of the e^+ and e^- angular distributions separately to exhibit the charge asymmetry. The numerical result displayed in Fig. 6 shows the limit of this statement. We suggest the measurement of two observables namely \mathcal{S} and \mathcal{A} .

We would like to emphasize that the Monte Carlo event generators have been developed in the course of this work. For this particular reaction, our code is more accurate than the PHOTOS Monte Carlo [26] where the interference between the initial and final state radiation is ignored.

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Appendix A: Box diagrams contribution

With the usual Mandelstam variables s , t and u of the process. θ is the scattering angle of the positron in the center of mass system.

$$\frac{d^2\sigma}{d\Omega_{e^+}} = \frac{\alpha^2}{4s\beta_p}(2 - \sin^2\theta) \left[1 + \frac{\alpha}{\pi} \frac{2I(s, t, u)}{s(2 - \sin^2\theta)} \right] \quad (\text{A1})$$

$$s(2 - \sin^2\theta) = \frac{2}{s} \left[t^2 + u^2 - 4M^2(t + u) + 6M^4 \right] \quad (\text{A2})$$

$$\delta_{box} = \frac{\alpha}{\pi} \frac{2I(s, t, u)}{s(2 - \sin^2\theta)} = \frac{\alpha}{\pi} \frac{sI(s, t, u)}{t^2 + u^2 - 4M^2(t + u) + 6M^4} \quad (\text{A3})$$

$$\Delta = t^2 + u^2 - 4M^2(t + u) + 6M^4 \quad (\text{A4})$$

$$\delta_{box} = \frac{\alpha}{\pi} \frac{s I(s, t, u)}{\Delta} \quad (\text{A5})$$

The five different terms in $I(s, t, u)$ are

$$I(s, t, u) = I^1(s, t, u) + I^2(s, t, u) + I^3(s, t, u) + I^4(s, t, u) + I^5(s, t, u)$$

$$sI^5(s, t, u) = 2 \Delta L_{tu} L_{M\lambda} + 2 \Delta L_{tu} L_s$$

$$\frac{sI^5(s, t, u)}{\Delta} = 2 L_{tu} L_{M\lambda} + 2 L_{tu} L_s$$

This last term contains the divergent part. Rewritting the δ_{box} in term of F_{box} , one gets

$$\delta_{box} = \frac{\alpha}{\pi} F_{box} \quad (\text{A6})$$

$$F_{box} = F_{1-box} + F_{2-box} + F_{3-box} + F_{4-box} + F_{5-box} \quad (\text{A7})$$

$$F_{1-box} = \frac{s(u-t)}{\Delta} \left[\left(\frac{2M^2}{\beta_p^2} + t + u \right) I_0 - \frac{\pi^2}{6} + \frac{1}{2} L_{\beta_p}^2 - \frac{L_{\beta_p}}{\beta_p^2} \right] \quad (\text{A8})$$

With

$$I_0 = \frac{1}{s\beta_p} \left\{ L_s L_{\beta_p} - \frac{1}{2} L_{\beta_p}^2 - \frac{\pi^2}{6} + 2Sp\left(\frac{1+\beta_p}{2}\right) - 2Sp\left(\frac{1-\beta_p}{2}\right) - 2Sp\left(-\frac{1-\beta_p}{1+\beta_p}\right) \right\} \quad (\text{A9})$$

and

$$F_{2-box} = \frac{s(2t+s)}{\Delta} \left[\frac{1}{2} L_{ts}^2 - Sp\left(\frac{-t}{M^2-t}\right) \right] \quad (\text{A10})$$

$$F_{3-box} = -\frac{s(2u+s)}{\Delta} \left[\frac{1}{2} L_{us}^2 - Sp\left(\frac{-u}{M^2-u}\right) \right] \quad (\text{A11})$$

$$F_{4-box} = \frac{s(ut - M^2(s + M^2))}{\Delta} \left[\frac{L_{ts}}{t} - \frac{L_{us}}{u} + \frac{u-t}{ut} L_s \right] \quad (\text{A12})$$

$$F_{5-box} = 2 L_{tu} L_s + 2 L_{tu} L_{M\lambda} \quad (\text{A13})$$

$$L_s = \ln \frac{s}{M^2} \quad L_{tu} = \ln \frac{M^2 - t}{M^2 - u} \quad L_{ts} = \ln \frac{M^2 - t}{s} \quad L_{us} = \ln \frac{M^2 - u}{s} \quad (\text{A14})$$

$$L_{\beta_p} = \ln \frac{1 + \beta_p}{1 - \beta_p} \quad L_{M\lambda} = \ln \frac{M^2}{\lambda^2} \quad (\text{A15})$$

Appendix B: 't Hooft and Veltman integrals

We need to estimate the integral of the type :

$$\frac{\mathcal{L}_{ij}}{2} \equiv \int_0^{\omega'} \frac{1}{(p_i \cdot k)(p_j \cdot k)} \frac{d^3 \mathbf{k}}{2k_0} \quad (\text{B1})$$

where $\omega' = \sqrt{\omega^2 - \lambda^2}$ and $\omega = (k_0)_{max}$ is the maximum energy of the emitted photon.

$$p = \eta p_i \quad q = p_j \quad (p - q)^2 = 0 \quad (\eta p_i - p_j)_0 \text{ and } p_{j0} \text{ with the same sign} \quad (\text{B2})$$

$$\eta^2 p_i^2 - 2\eta p_i \cdot p_j + p_j^2 = 0 \quad (\text{B3})$$

$$\ell = p_0 - q_0 \quad v = \frac{p^2 - q^2}{2\ell} \quad (\text{B4})$$

$$\mathcal{L}_{ij}(\text{div}) = 2\pi \frac{\eta}{v\ell} \ln \frac{p^2}{q^2} \ln \frac{2\omega}{\lambda} \quad (\text{B5})$$

$$\begin{aligned} \mathcal{L}_{ij}(\text{finite}) = & 2\pi \frac{\eta}{v\ell} \left[\frac{1}{4} \ln^2 \frac{p_0 - |\mathbf{p}|}{p_0 + |\mathbf{p}|} - \frac{1}{4} \ln^2 \frac{q_0 - |\mathbf{q}|}{q_0 + |\mathbf{q}|} \right] \\ & + 2\pi \frac{\eta}{v\ell} \left[Sp\left(1 - \frac{p_0 + |\mathbf{p}|}{v}\right) - Sp\left(1 - \frac{q_0 + |\mathbf{q}|}{v}\right) \right] \\ & + 2\pi \frac{\eta}{v\ell} \left[Sp\left(1 - \frac{p_0 - |\mathbf{p}|}{v}\right) - Sp\left(1 - \frac{q_0 - |\mathbf{q}|}{v}\right) \right] \end{aligned} \quad (\text{B6})$$

Appendix C: Hard photon contribution

Defining

$$\Gamma_{NN\gamma}^\sigma(k) = F_{10}(k^2)\gamma^\sigma - \frac{F_{20}(k^2)}{4M}(k\gamma^\sigma - \gamma^\sigma k) \quad F_{10}(k^2) = 1 \quad F_{20}(k^2) = \kappa_p \quad (\text{C1})$$

$$q' = p^- + p^+ - k = q - k \quad (\text{C2})$$

$$\mathcal{M}_1(\lambda, m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) = \frac{A_1^\sigma(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p)}{D_1} \varepsilon_\sigma^*(k, \lambda) \quad (\text{C3})$$

$$A_1^\sigma(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) \equiv \frac{i}{q^2} e_p e_{e^-}^2 \left[\bar{v}_{\bar{p}} \Gamma_{NN\gamma}^\mu(q) u_p \right] \left[\bar{u}_{e^-} \gamma_\mu (-\not{k} - \not{q}^+ + m) \gamma^\sigma v_{e^+} \right] \quad (\text{C4})$$

$$D_1 \equiv (k + q^+)^2 - m^2 = 2 k \cdot q^+ \quad (\text{C5})$$

$$\mathcal{M}_2(\lambda, m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) = \frac{A_2^\sigma(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p)}{D_2} \varepsilon_\sigma^*(k, \lambda) \quad (\text{C6})$$

$$A_2^\sigma(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) \equiv \frac{i}{q^2} e_p e_{e^-}^2 \left[\bar{v}_{\bar{p}} \Gamma_{NN\gamma}^\mu(q) u_p \right] \left[\bar{u}_{e^-} \gamma^\sigma (\not{k} + \not{q}^- + m) \gamma_\mu v_{e^+} \right] \quad (\text{C7})$$

$$D_2 \equiv (k + q^-)^2 - m^2 = 2 k \cdot q^- \quad (\text{C8})$$

$$\mathcal{M}_3(\lambda, m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) = \frac{A_3^\sigma(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p)}{D_3} \varepsilon_\sigma^*(k, \lambda) \quad (\text{C9})$$

$$A_3^\sigma(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) \equiv \frac{i}{q'^2} e_p^2 e_{e^-} \left[\bar{v}_{\bar{p}} \Gamma_{NN\gamma}^\sigma(k) (\not{k} - \not{p}^- + M) \Gamma_{NN\gamma}^\mu(q') u_p \right] \left[\bar{u}_{e^-} \gamma_\mu v_{e^+} \right] \quad (\text{C10})$$

$$D_3 \equiv (p^- - k)^2 - M^2 = -2 k \cdot p^- \quad (\text{C11})$$

$$\mathcal{M}_4(\lambda, m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) = \frac{A_4^\sigma(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p)}{D_4} \varepsilon_\sigma^*(k, \lambda) \quad (\text{C12})$$

$$A_4^\sigma(m_{e^+}, m_{e^-}; m_{\bar{p}}, m_p) \equiv \frac{i}{q'^2} e_p^2 e_{e^-} \left[\bar{v}_{\bar{p}} \Gamma_{NN\gamma}^\mu(q') (\not{p}^+ - \not{k} + M) \Gamma_{NN\gamma}^\sigma(k) u_p \right] \left[\bar{u}_{e^-} \gamma_\mu v_{e^+} \right] \quad (\text{C13})$$

$$D_4 \equiv (p^+ - k)^2 - M^2 = -2 k \cdot p^+ \quad (\text{C14})$$

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